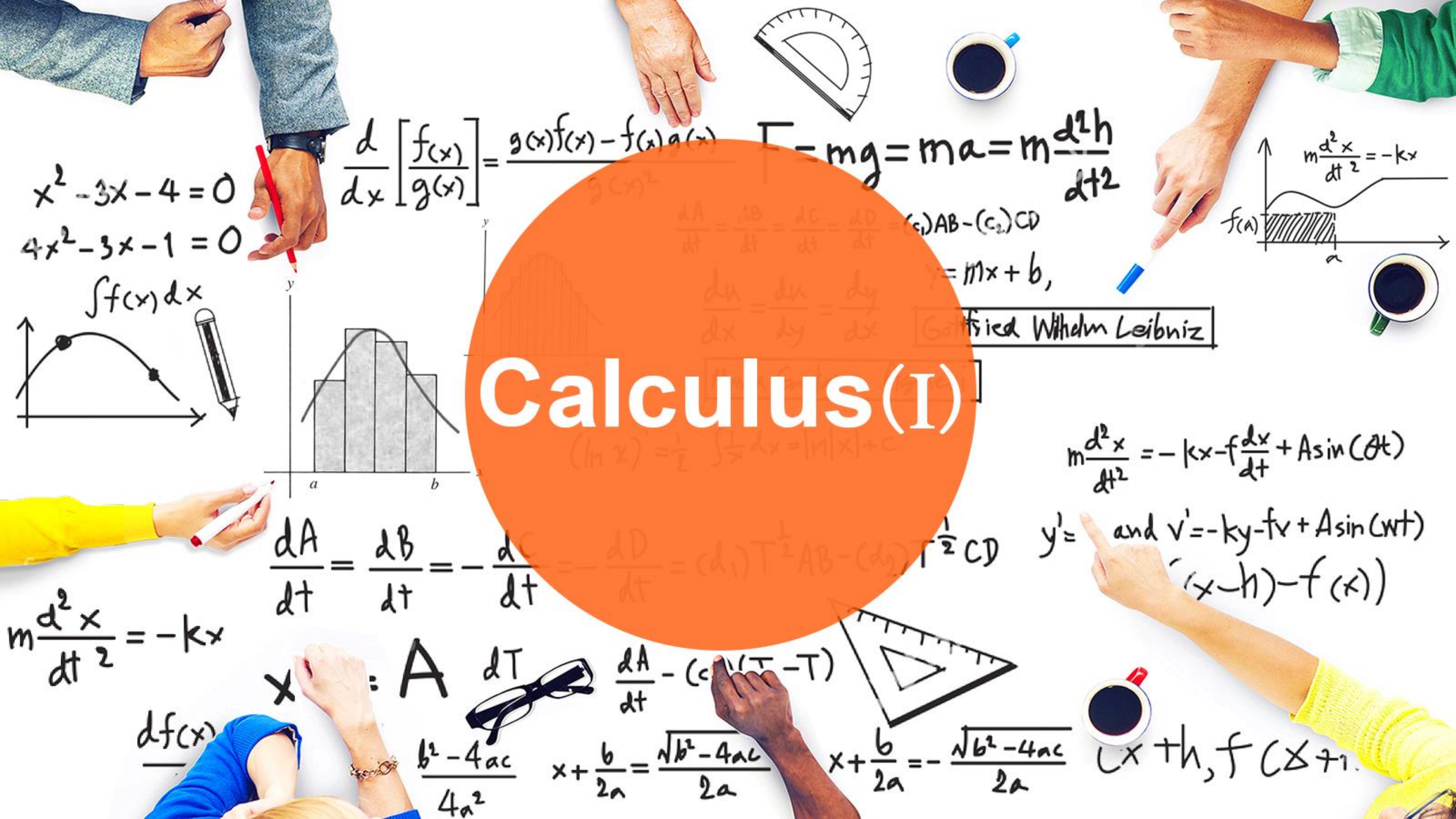
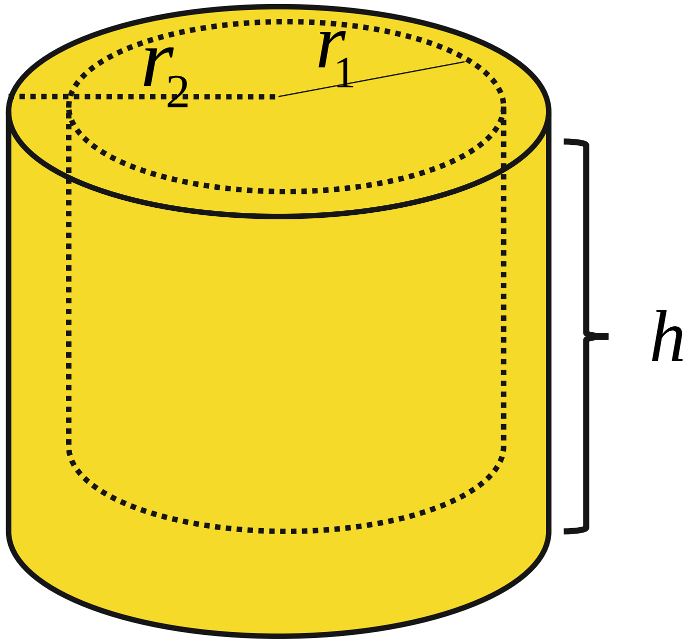

Calculus(I)

Volumes of Solids （Shell Method）

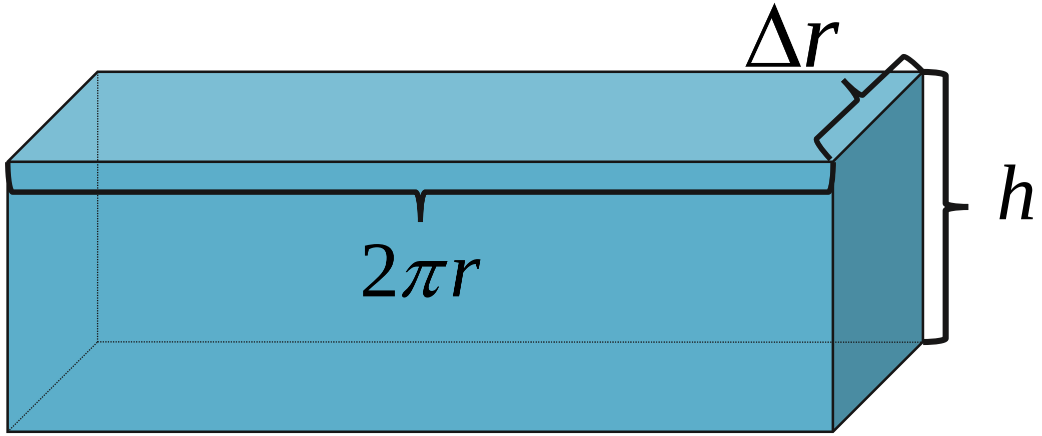
Lecturer: Xue Deng

How to compute the left shell's volume?



$$
\begin{aligned}
V &= \text{(the area of base)(height)} \\
&= (\pi r_2^{\,2} - \pi r_1^{\,2})h \\
&= \pi(r_2 - r_1)(r_2 + r_1)h \\
&= 2\pi\left(\frac{r_2 + r_1}{2}\right)h(r_2 - r_1)
\end{aligned}
$$

How to compute the left shell's volume?



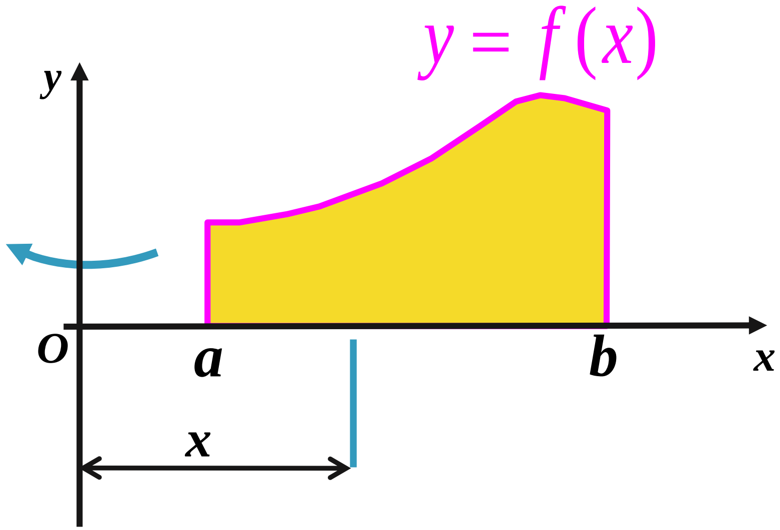
$$V = 2\pi(\text{average radius})(\text{height})(\text{thicknes})$$

$$\text{average radius} = \frac{r_2 + r_1}{2}, \quad \text{height} = h, \quad \text{thicknes} = \Delta r$$

$$= 2\pi r h \Delta r.$$

Volumes of Solids （Shell Method）

Q:If we revolve the fowling figure about y-axis, it will generate a revolution ,then, the volume of the shell?



$$\Delta V \approx 2\pi x f(x)\Delta x,$$

$$V = 2\pi \int_a^b x f(x) \mathrm{d}x.$$

Example 1

The region bounded by $y = \frac{1}{\sqrt{x}}$, the x-axis, $x = 1$ and $x = 4$ is revolved about y-axis, find the volume of the resulting solid.

$$\Delta V \approx 2\pi x f(x) \Delta x,$$

$$\Delta V \approx 2\pi x \frac{1}{\sqrt{x}} \Delta x,$$

$$V = 2\pi \int_1^4 \sqrt{x}\, dx$$

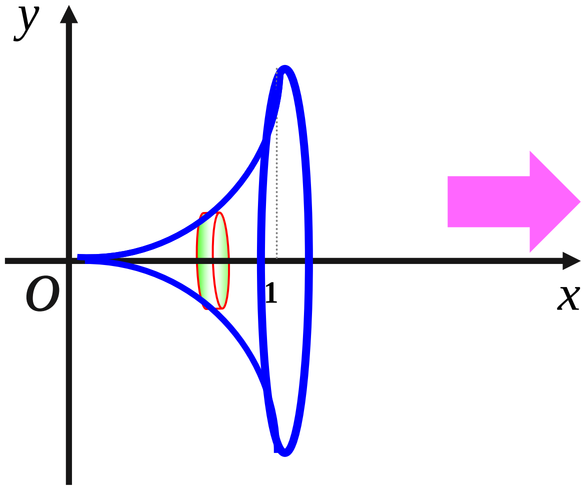
$$= 2\pi \frac{2}{3} x^{\frac{3}{2}} \Big|_1^4$$

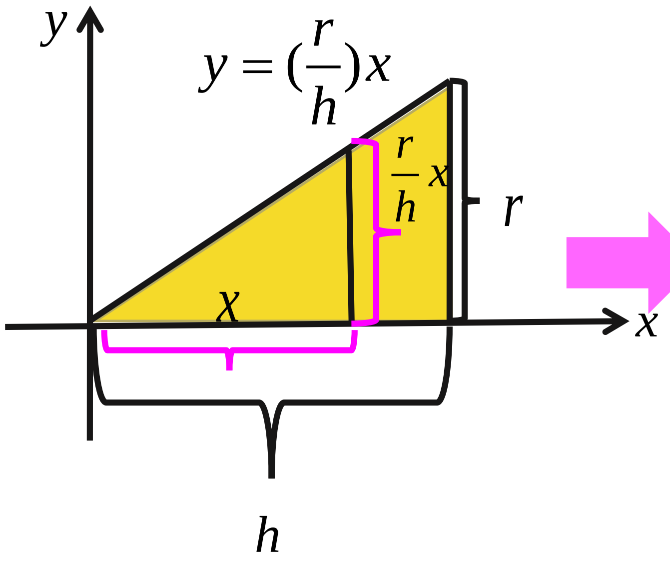
$$= \frac{28\pi}{3}.$$

Example 2

The region bounded by the line $y=(\frac{r}{h})x$, x-axis, $x=h$ is revolved about the x-axis there by generated a core($r>0, h>0$), find its volume by the shell method.

(Recall: Method 1-Disk Method) $V=\int_a^b \pi[f(x)]^2 \mathrm{d}x$





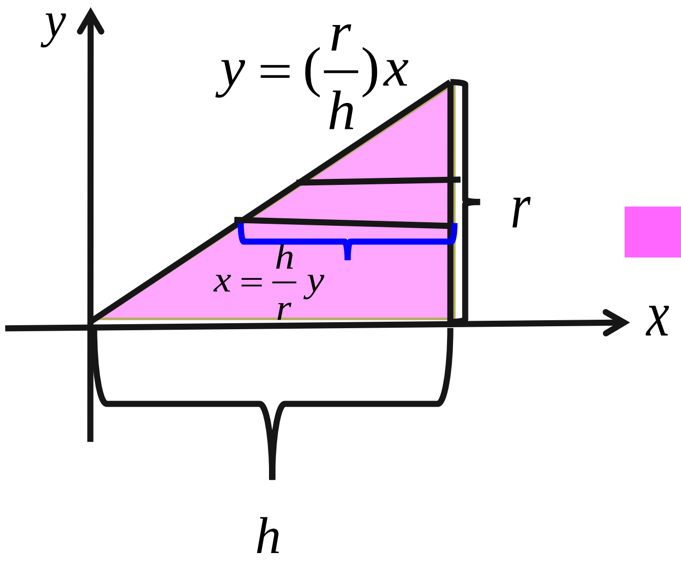
$$V=\pi\int_0^h \frac{r^2}{h^2}x^2\mathrm{d}x$$

$$=\pi\frac{r^2}{h^2}\int_0^h x^2\mathrm{d}x$$

$$=\pi\frac{r^2}{h^2}\left[\frac{x^3}{3}\right]_0^h=\frac{\pi r^2 h}{3}.$$

Examples 2

(Method 2-Shell Method)



$$\Delta V \approx 2\pi y(h - \frac{h}{r}y)\Delta x,$$

$$V = 2\pi \int_0^r y(h - \frac{h}{r}y)\mathrm{d}x.$$

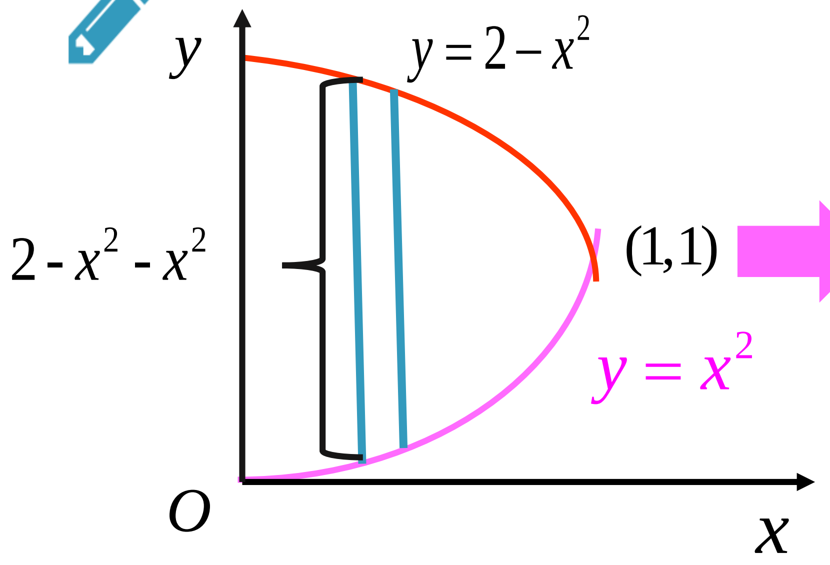
$$\Delta V = 2\pi y(h - \frac{h}{r}y)\Delta y,$$

$$V = 2\pi \int_0^r y(h - \frac{h}{r}y)\mathrm{d}y$$

$$= \frac{\pi r^2 h}{3}.$$

Example 3

Find the volume of the solid generated by revolving the region in the first quadrant that is above the parabola $y = x^2$ and below the parabola $y = 2 - x^2$ about the y-axis.



$$\Delta V \approx 2\pi x(2 - x^2 - x^2)\Delta x,$$

$$V = 2\pi \int_0^1 x(2 - 2x^2)\mathrm{d}x.$$

$$\Delta V = 2\pi x(2 - 2x^2)\Delta x,$$

$$V = 2\pi \int_0^1 x(2 - 2x^2)\mathrm{d}x$$

$$= \pi.$$

Summary of Shell Method

1. $$\Delta V \approx 2\pi x f(x)\Delta x$$

2. $$V = 2\pi \int_a^b x f(x)\mathrm{d}x$$

Questions and Answers

Q: Set up and evaluate an integral for the volume of the solid that results when the region R abound by $y = 3 + 2x - x^2$, $x \geq 0$ and $y \geq 0$ is revolved about y - $axis$.

$$\Delta V \approx 2\pi x(3 + 2x - x^2)\Delta x,$$

$$V = 2\pi \int_0^3 x(3 + 2x - x^2)\mathrm{d}x.$$

$$V = 2\pi \int_0^3 x(3 + 2x - x^2)\mathrm{d}x.$$

$$= \frac{45}{2}\pi.$$

Volumes of Solids （Disk Method）

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