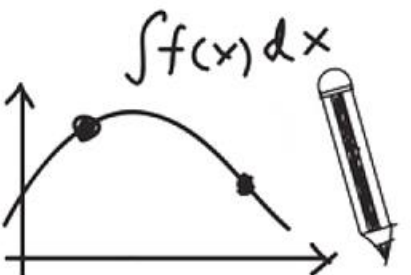


$$x^2 - 3x - 4 = 0$$
$$4x^2 - 3x - 1 = 0$$

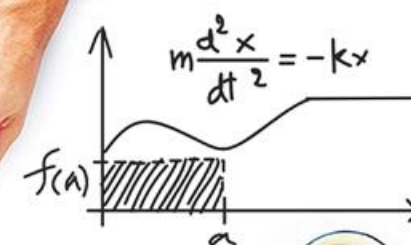


$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$



# Calculus(I)

$$F = mg = ma = m \frac{d^2h}{dt^2}$$



Gottfried Wilhelm Leibniz

$$\frac{dA}{dt} = \frac{dB}{dt} = -\frac{dC}{dt} = \frac{dD}{dt} = (c_1)T^{\frac{1}{2}}AB - (c_2)T^{\frac{1}{2}}CD$$

$$m \frac{d^2x}{dt^2} = -kx - f \frac{dx}{dt} + A \sin(\omega t)$$
$$y' = \text{and } v' = -ky - fv + A \sin(\omega t)$$

$$m \frac{d^2x}{dt^2} = -kx$$

$$x = A \frac{dT}{dt} - (c_1)(T - T)$$

$$\frac{df(x)}{dx}$$

$$\frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$cx + h, f(x) + i$$

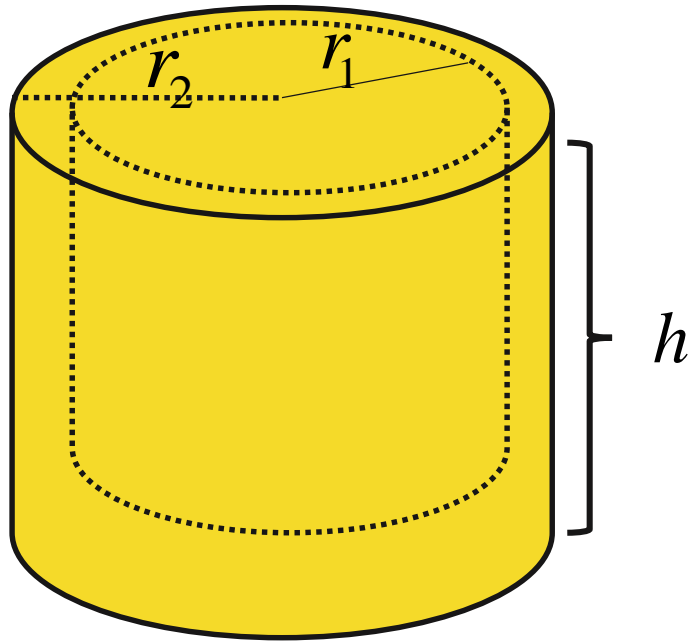


# Volumes of Solids (Shell Method)

Lecturer: Xue Deng



# How to compute the left shell's volume?



$$V = (\text{the area of base})(\text{height})$$

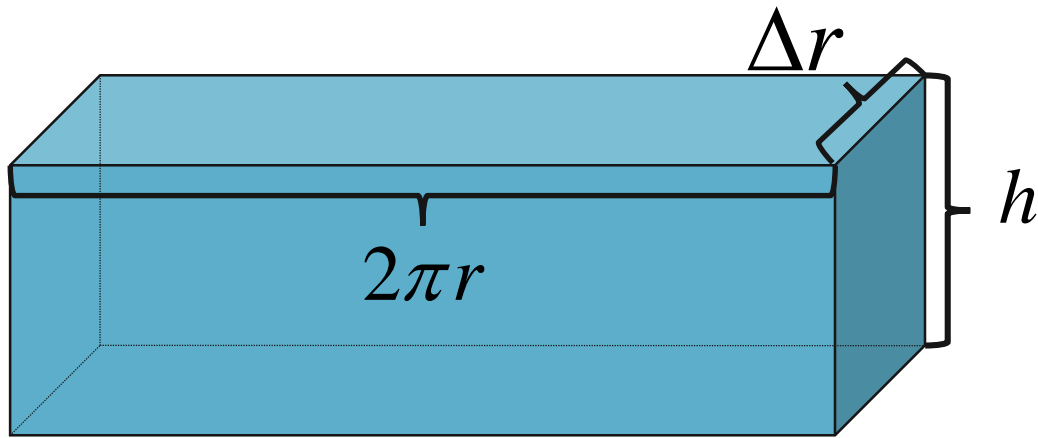
$$= (\pi r_2^2 - \pi r_1^2)h$$

$$= \pi(r_2 - r_1)(r_2 + r_1)h$$

$$= 2\pi\left(\frac{r_2 + r_1}{2}\right)h(r_2 - r_1)$$



# How to compute the left shell's volume?

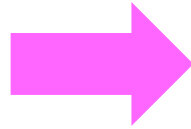
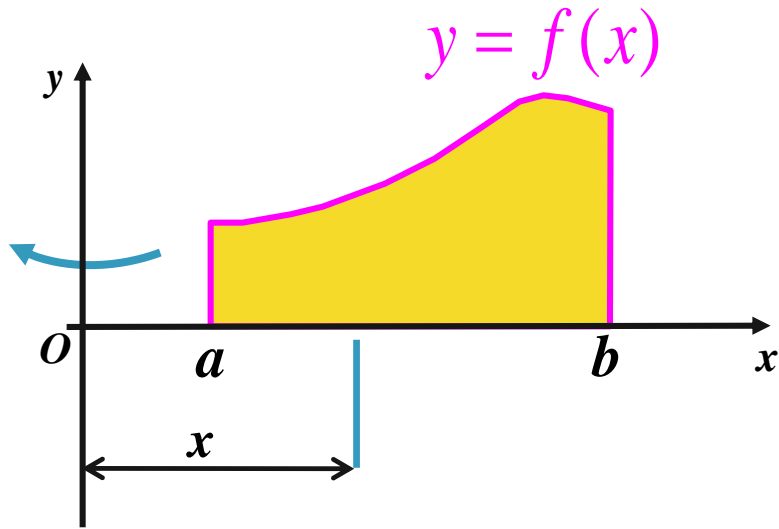


$V = 2\pi \left( \frac{r_2 + r_1}{2} \right) (h) (\Delta r)$   
 $= 2\pi r h \Delta r.$

*(Note: The diagram includes pink annotations: a box around  $\frac{r_2 + r_1}{2}$ , a box around the text "(average radius)", a box around "(height)", a box around "(thickness)", and a box around  $\Delta r$ . Arrows point from the boxes to the corresponding terms in the equation.)*

# Volumes of Solids (Shell Method)

Q: If we revolve the following figure about y-axis, it will generate a revolution, then, the volume of the shell?



$$\Delta V \approx 2\pi x f(x) \Delta x,$$

$$V = 2\pi \int_a^b x f(x) dx.$$

# Example 1

The region bounded by  $y = \frac{1}{\sqrt{x}}$ , the  $x$ -axis,  $x = 1$  and  $x = 4$  is revolved about  $y$ -axis, find the volume of the resulting solid.



$$\Delta V \approx 2\pi x f(x) \Delta x,$$

$$\Delta V \approx 2\pi x \frac{1}{\sqrt{x}} \Delta x,$$

$$V = 2\pi \int_1^4 \sqrt{x} dx$$

$$= 2\pi \frac{2}{3} x^{\frac{3}{2}} \Big|_1^4$$

$$= \frac{28\pi}{3}.$$

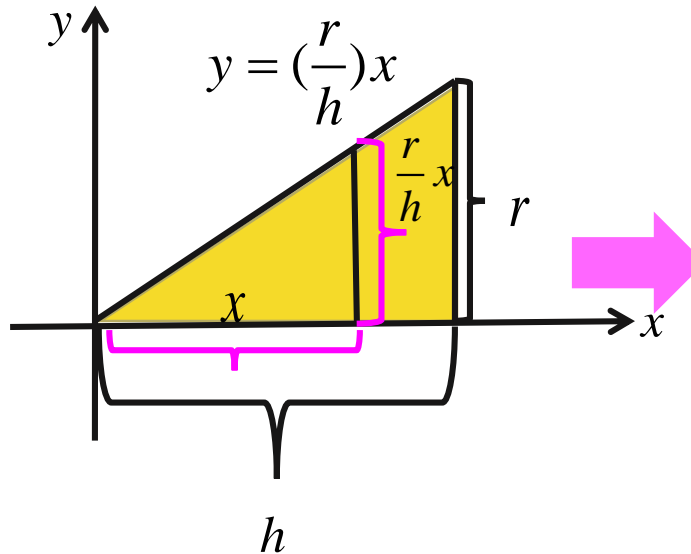
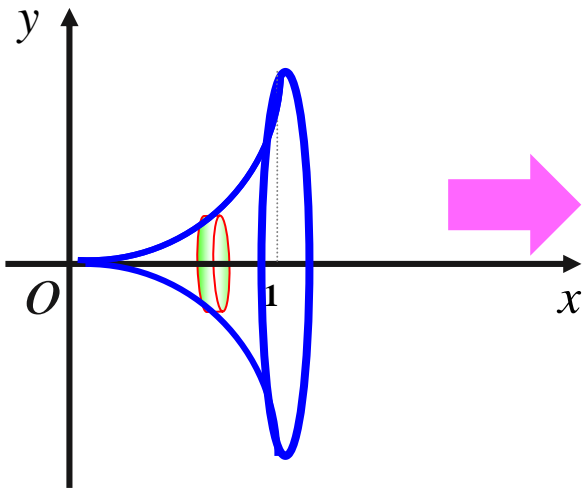
# Example 2

The region bounded by the line  $y = \left(\frac{r}{h}\right)x$ ,  $x$ -axis,  $x = h$  is revolved about the  $x$ -axis there by generated a core ( $r > 0, h > 0$ ), find its volume by the shell method.



(Recall: **Method 1-Disk Method**)

$$V = \int_a^b \pi [f(x)]^2 dx$$

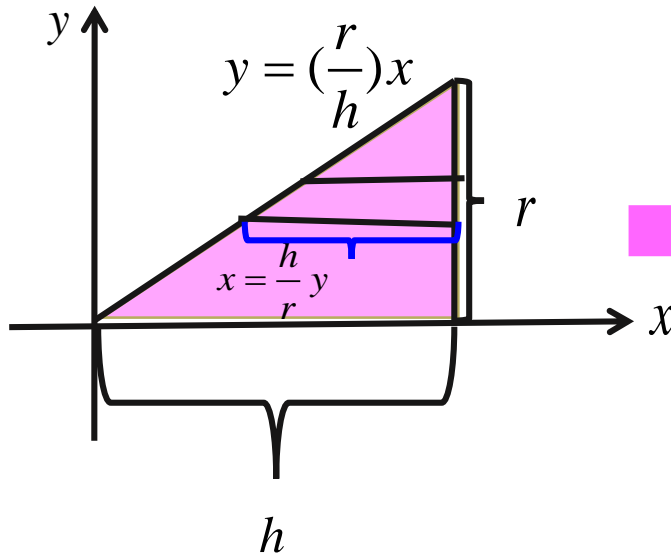


$$\begin{aligned} V &= \pi \int_0^h \frac{r^2}{h^2} x^2 dx \\ &= \pi \frac{r^2}{h^2} \int_0^h x^2 dx \\ &= \pi \frac{r^2}{h^2} \left[ \frac{x^3}{3} \right]_0^h = \frac{\pi r^2 h}{3}. \end{aligned}$$

# Examples 2



(Method 2-Shell Method)



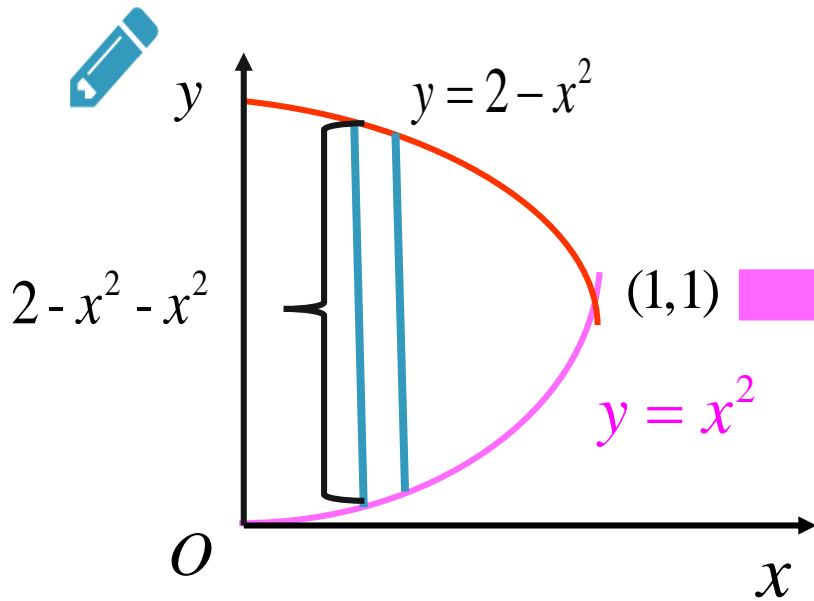
$$\Delta V \approx 2\pi y\left(h - \frac{h}{r}y\right)\Delta x,$$
$$V = 2\pi \int_0^r y\left(h - \frac{h}{r}y\right)dy.$$

$$\Delta V = 2\pi y\left(h - \frac{h}{r}y\right)\Delta y,$$
$$V = 2\pi \int_0^r y\left(h - \frac{h}{r}y\right)dy$$
$$= \frac{\pi r^2 h}{3}.$$



# Example 3

Find the volume of the solid generated by revolving the region in the first quadrant that is above the parabola  $y = x^2$  and below the parabola  $y = 2 - x^2$  about the  $y$ -axis.



$$\Delta V \approx 2\pi x(2 - x^2 - x^2)\Delta x,$$

$$V = 2\pi \int_0^1 x(2 - 2x^2) dx.$$

$$\Delta V = 2\pi x(2 - 2x^2)\Delta x,$$

$$V = 2\pi \int_0^1 x(2 - 2x^2) dx$$

$$= \pi.$$

# Summary of Shell Method

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1

$$\Delta V \approx 2\pi x f(x) \Delta x$$

2

$$V = 2\pi \int_a^b x f(x) dx$$

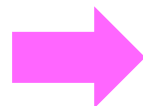
# Questions and Answers

Q: Set up and evaluate an integral for the volume of the solid that results when the region  $R$  bounded by  $y = 3 + 2x - x^2$ ,  $x \geq 0$  and  $y \geq 0$  is revolved about  $y$ -axis.



$$\Delta V \approx 2\pi x(3 + 2x - x^2)\Delta x,$$

$$V = 2\pi \int_0^3 x(3 + 2x - x^2)dx.$$



$$V = 2\pi \int_0^3 x(3 + 2x - x^2)dx.$$

$$= \frac{45}{2}\pi.$$

# Volumes of Solids (Disk Method)

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# Volumes of Solids (Shell Method)

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